SYMBOLIC DYNAMICS FROM BIOLOGICAL TIME SERIES

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parameters the sequence of symbols may be a zeroth order Markov process (Bernoulli process [5]). In other words, a deterministic system described by three coupled nonlinear differential equations of the first order may generate a sequence of totally uncorrelated symbols, as if they were generated by a stochastic process such as the coin tossing. Nicolis et al. [6,7] considered the three-variable Rössler model in the chaotic region. Each time a variable crosses a prescribed threshold, a symbol specific to that variable is produced. The sequence of these symbols will then provide the image of the various thresholds encountered successively by the dynamics. They showed that the associated Markov process is at least of the fifth order. At the opposite of the Lorenz model, the

\[
P^{(1)}(X_1, X_2, X_3, \ldots X_L) = P(X_1) P(X_2 | X_1) P(X_3 | X_2) \ldots P(X_L | X_{L-1}) ,
\]

\[
P^{(2)}(X_1, X_2, X_3, \ldots X_L) = P(X_1) P(X_2 | X_1) \times P(X_3 | X_1, X_2) P(X_L | X_{L-2}, X_{L-1}) ,
\]

etc.

The values \( P(X_1, X_2, X_3, \ldots X_L) \) deduced from counting are compared with the computed values of \( P^{(k)}(X_1, X_2, X_3, \ldots X_L) \) by using a test statistic such as

\[
\chi^2 = \frac{1}{n L} \sum_{i=1}^{n L} (P_i - P_i^{(k)})^2 / P_i .
\]
a function of $N$, the same function of $N$ is seen in $\chi^2$ for $k \geq k_0$ while for $k < k_0$, $\chi^2$ remains independent of $N$. This property can be used to discriminate between the two cases, providing a criterion to estimate $\alpha$ for $k \geq k_0$ ($\alpha = 0.51 \pm 0.05$ for $k = 3$ to 5), therefore our conjecture (7) seems to hold in this case. A similar scaling was found for the tent map and the logistic map ($r = 4$). The order estimated was reasonable.
Fig. 2. (a) Electrical activity of the heart (ECG) recorded at the level of the thorax from a normal human subject. (b) Electrical activity of the human brain (EEG) during a pathological state. Both signals exhibit a repetition of the same pattern at irregular intervals.

all point to the same conclusion, namely that the electrical activity of the heart as well as the brain waves may follow chaotic dynamics.

From fig. 2, it is seen that both for ECG and EEG recordings, electric potential spikes are produced at irregular time intervals. In the case of the EEG, the distribution of inter-spike time intervals may be bi-modal. Associating a symbol to each peak of the dis-
is of $k_0=3$ for ECG intervals (see fig. 3c for comparison). In the case of the EEG intervals, $\chi^2$ converges to zero at $k_0=6$ (fig. 3b). The convergence of $\chi^2$ to zero does not depend significantly on the number $n$ of symbols used. Actually, different values of $n$ were tested (from $n=2$ to $n=5$) and give similar results. It is remarkable that the order of the Markov process obtained from the symbolic dynamics generated with "short" and "long" intervals does not differ significantly. Finally, a dynamics associated with the attractor and the order of the time interval Markov chain has not yet been established. A high order Markov process may be reminiscent of chaotic dynamics, such as for example in the Rössler model. More generally it has been shown [16] that a given class of chaotic dynamical systems can be mapped into a well defined stochastic process described by a master equation. In some cases, this process can be of higher order.
ECG (3 pseudo-cycles), the autocorrelation function vanishes more rapidly for the EEG ($t_c = 2.6$ s or 3.8 pseudo-cycles) than for the ECG ($t_c = 5$ s or 4.4 pseudo-cycles). This result shows that symbolic dynamics may bring complementary information to usual techniques.

The symbolic analysis performed here is independent of any phase space reconstruction algorithm, and does not suffer from the limitations inherent to the reconstruction of high dimensional attractors [22,23]. Moreover, the intervals are subject to a coarse “digitization” into a few symbols. Therefore, we expect that the influence of additive noise will be insignificant (the amplitude of the noise is usually much smaller than the typical order magnitude of the partition of intervals).

In conclusion, in this paper we have outlined a method, based on standard statistical concepts, to evaluate the order of a Markov chain obtained from a time series. The scaling properties of the $\chi^2$ test allows the accurate determination of the order of a process described by a sufficiently long sequence of symbols ($\sim 10^5$). However in experimental situations, the number of symbols is usually smaller ($\sim 10^3$) and in that case the order can still be estimated without reference to the scaling criterion.

The two biological signals considered in this context exhibit features allowing the transformation of the variables into symbols, namely that the dynamics of both rhythms share a common property: a seemingly irregular succession of the same type of oscillations constitutes their time behavior. We find that this succession is nevertheless temporally correlated rather than driven by noise. These results must be put in parallel with previous work [12–14,18–21] which provided evidences that the aperiodic time evolution of these important physiological states stems from chaotic dynamics.

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References