SYMBOLOIC DYNAMICS FROM BIOLOGICAL TIME SERIES

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A method is outlined for estimating the order of a Markov process, and applied to both model and experimental time series. Unexpected high values are seen for heart and brain recordings, showing that for these systems the succession of time intervals between patterns exhibits intimate correlations.

Chaotic dynamical systems share many common properties with stochastic processes. For example, the divergence of nearby trajectories imposes a limit on the predictability of the time behavior of the system beyond a characteristic time which is of the order of the inverse of the Kolmogorov entropy [1,2]. The deterministic description of the attractor as a set of trajectories in the phase space can then be replaced by a statistical description in terms of a probability density in the phase space [2].

On the other hand, chaotic dynamics contains remarkable regularities that are not usually seen in noise-driven systems. The same regions of the phase space may be visited at various time during the dynamical evolution of the system. In some cases, the system's dynamics may consist of an irregular succession of distinguishable events corresponding to a visit of the trajectory in a well defined region of the phase space. For example in the Lorenz model [3], the chaotic attractor is composed of two identical regions, symmetrically disposed around the z-axis. Despite the deterministic character of the Lorenz equations, these regions are visited by the trajectories in a stochastic manner [4].

In order to describe the succession of events in such systems, it is useful to associate a symbol to each one of these events or, in other words, to partition the phase space into a finite set of regions each associated with a different symbol. This operation transforms the dynamics of continuous variables into a discrete sequence of symbols whose statistical properties and time correlations can be studied. In particular, it is of interest to consider such a sequence of symbols as a Markov chain. The order of the corresponding Markov process will give useful information about the time correlations of the underlying dynamical system.

In this paper, we will show how to use the symbolic description to assess information about the dynamics of an experimental system, usually known through a single time series representing the evolution of one of its variables. First, we will describe a method, based on standard statistical concepts, to evaluate the order of a Markov chain with a good accuracy. This method will be illustrated on two model systems, then it will be used to analyze the sequence of symbols extracted from two biological signals recorded from the human heart and the human brain. In these systems, the dynamics is made of the succession of a given pattern of activity and the time intervals between these patterns will be "translated" into symbols proportional to their duration. The sequence obtained will reflect the succession of intervals lengths and the order of the Markov chain will provide an interesting insight into the correlations among successive patterns.

For the Lorenz attractor, Aizawa [4] associated the letter L (respectively R) to each orbit if the trajectory was on the left (respectively right) side of the attractor. The sequence of symbols reflects the succession of visits of the trajectory in either side of the attractor. He showed that for a given set of pa-
parameters the sequence of symbols may be a zeroth order Markov process (Bernoulli process [5]). In other words, a deterministic system described by three coupled nonlinear differential equations of the first order may generate a sequence of totally uncorrelated symbols, as if they were generated by a stochastic process such as the coin tossing. Nicolis et al. [6, 7] considered the three-variable Rössler model in the chaotic region. Each time a variable crosses a prescribed threshold, a symbol specific to that variable is produced. The sequence of these symbols will then provide the image of the various thresholds encountered successively by the dynamics. They showed

\[ P^{(1)}(X_1, X_2, X_3, \ldots X_L) = P(X_1)P(X_2 | X_1)P(X_3 | X_2) \ldots P(X_L | X_{L-1}) \]  

(3)

\[ P^{(2)}(X_1, X_2, X_3, \ldots X_L) = P(X_1)P(X_2 | X_1) \times P(X_3 | X_1, X_2)P(X_L | X_{L-2} X_{L-1}) \]  

(4)

etc.

The values \( P(X_1, X_2, X_3, \ldots X_L) \) deduced from counting are compared with the computed values of \( P^{(k)}(X_1, X_2, X_3, \ldots X_L) \) by using a test statistic such as

\[ \sum_{i=1}^{n} \left( \frac{p_{obs} - p_{comp}}{p_{comp}} \right)^2 / p_{comp} \]  

(5)
is of $k_0=3$ for ECG intervals (see fig. 3c for comparison). In the case of the EEG intervals, $\chi^2$ converges to zero at $k_0=6$ (fig. 3b). The convergence of $\chi^2$ to zero does not depend significantly on the number $n$ of symbols used. Actually, different values of $n$ were tested (from $n=2$ to $n=5$) and give similar results. It is remarkable that the order of the Markov process obtained from the symbolic dynamics generated with “short” and “long” intervals does not differ significantly from the dynamics generated with a different subdivision of the intervals (“very short”, “short”, “long” and “very long” for example).

The values of $k_0$ were computed from a sequence of length $N=2132$ (ECG) and $N=1756$ (EEG). These values are too small to use the scaling criterion described above. It is to be noticed that these lengths correspond indeed to exceptionally long time series (610000 points and 305000 points, representing a total time of respectively 2440 s and 1220 s). We verified the stationarity of these signals from the evaluation of recurrence plots [15]. For both cases, several recording sites were available and the ECG sequences were tested from three other individuals ($N=925$, $N=929$, $N=1224$ respectively). In all cases, for different word lengths $n^1$, the same results were observed, namely that the order of ECG intervals succession is at least three (a fourth order has been observed for one of the three other subjects).

One of the most remarkable facts is that these sequences produce only few words with high frequency while some other words never appear. Among all the $2^n$ possible words of length $L=9$ ($n=2$), only 279 were realized by the dynamics of ECG intervals and only 151 words were seen in EEG sequences. This finding shows that despite the apparent irregularity of these symbol sequences, strong “grammatical rules” may exist. These rules probably reflect the dynamics of the attractor and the order of the time interval Markov chain has not yet been established, a high order Markov process may be reminiscent of chaotic dynamics, such as for example in the Rössler model. More generally it has been shown [16] that a given class of chaotic dynamical systems can be mapped into a well defined stochastic process described by a master equation. In some cases, this process can be of higher order.

The fact that the cardiac beat-to-beat intervals may be described by a third order Markov process shows that the heart rhythm is not a limit cycle perturbed by random noise. The irregular variations of the R–R intervals are well known by physiologists and here we show that the successive variations of the rhythm are strongly correlated from one beat to the next. Furthermore, we find a correlation length (or “memory”) of three beats and this fact is compatible with our previous finding that the heart might follow chaotic dynamics [12]. By an independent graphical method, we had also found strong correlations between three consecutive intervals [12].

The sequence of EEG intervals is associated with a very high order Markov process in the case we have studied here. Despite the apparent aperiodicity of this rhythmic activity, the intervals between spikes are strongly correlated. This result must be put in parallel with the exceptionally low correlation dimension observed for this phenomenon [13] compared to the normal brain states which are of higher dimensionality [18–21] [3]. We found that four degrees of freedom are sufficient to describe the dynamics of the brain in the above described state [13] and the variations of the rhythm are probably of chaotic nature. Here again, a very strong correlation between intervals is what we could expect from such a deterministic system.
ECG (3 pseudo-cycles), the autocorrelation function vanishes more rapidly for the EEG ($t_c = 2.6$ s or 3.8 pseudo-cycles) than for the ECG ($t_c = 5$ s or 4.4 pseudo-cycles). This result shows that symbolic dynamics may bring complementary information to usual techniques.

The symbolic analysis performed here is independent of any phase space reconstruction algorithm, and does not suffer from the limitations inherent to the reconstruction of high dimensional attractors [22,23]. Moreover, the intervals are subject to a

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References


