Conductance-Based Integrate-and-Fire Models

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A conductance-based model of Na\(^+\) and K\(^+\) currents underlying action potential generation is introduced by simplifying the quantitative model of Hodgkin and Huxley (HH). If the time course of rate constants can be approximated by a pulse, HH equations can be solved analytically. Pulse-based (PB) models generate action potentials very similar to the HH model but are computationally faster. Unlike the classical integrate-and-fire (IAF) approach, they take into account the changes of conductances during and after the spike, which have a determinant influence in shaping neuronal responses. Similarities and differences among PB, IAF, and HH models are illustrated for three cases: high-frequency repetitive firing, spike timing following random synaptic inputs, and network behavior in the presence of intrinsic currents.
An alternative approach was to simplify the process of action potential generation by representing the neuron as an integrate-and-fire (IAF) device (see Arbib, 1995). In this case, the neuron produces a spike when its membrane potential crosses a given threshold value and the membrane is instantaneously reset to its resting level. Although this type of model has been shown to be useful in computational neuroscience, it is important to note that it is a simplified representation of the complex dynamics of real neurons.
Figure 1: Comparison of action potential generation in Hodgkin-Huxley and pulse-based models. A train of spikes was generated by injecting a depolarizing current pulse (2 nA) in a single compartment model (area of 15,000 $\mu$m$^2$; $\tilde{g}_{k,j} = 30$ ms/cm$^2$; other parameters as in equation 2.1). The time course of the rate constants ($\alpha_m$, $\beta_m$, $\alpha_h$, $\beta_h$, $\alpha_n$, and $\beta_n$) and state variables ($m$, $h$, and $n$) is shown for each panel. In the Hodgkin-Huxley model (left panel), rate constants undergo sharp transitions when a spike occurs. Approximating these
given threshold. For example, \( \alpha_m \) will be assigned a positive value during the pulse and will be zero otherwise. Generalizing for other rate constants, before and after the pulse, we have:

\[
\alpha_m = 0, \quad \beta_m = \beta_M \\
\alpha_h = \alpha_H, \quad \beta_h = 0 \\
\alpha_n = 0, \quad \beta_n = \beta_N
\]

(2.2)

whereas during the pulse:

\[
\alpha_m = \alpha_M, \quad \beta_m = 0 \\
\alpha_h = 0, \quad \beta_h = \beta_H \\
\alpha_n = \alpha_N, \quad \beta_n = 0.
\]

(2.3)

Here, \( \alpha_M, \beta_M, \alpha_H, \beta_H, \alpha_N, \) and \( \beta_N \) are constant values. They are estimated from the value of the voltage-dependent expressions at hyperpolarized and depolarized potentials. Choosing \(-70\) mV and \(+20\) mV and using Traub and Miles's formulation gives:

\[
\alpha_M = \alpha_m(20) \approx 22 \text{ ms}^{-1} \\
\beta_M = \beta_m(-70) \approx 13 \text{ ms}^{-1} \\
\alpha_H = \alpha_h(-70) \approx 0.5 \text{ ms}^{-1} \\
\beta_H = \beta_h(20) \approx 4 \text{ ms}^{-1} \\
\alpha_N = \alpha_n(20) \approx 2.2 \text{ ms}^{-1} \\
\beta_N = \beta_n(-70) \approx 0.76 \text{ ms}^{-1}.
\]

(2.4)

It is then straightforward to solve equations 2.1, leading to the following expressions: before and after the pulse, the rate constants are given by:

\[
m(t) = m_0 \exp[-\beta_M(t-t_0)] \\
h(t) = 1 + (h_0 - 1) \exp[-\alpha_H(t-t_0)] \\
n(t) = n_0 \exp[-\beta_N(t-t_0)]
\]

(2.5)

where \( t_0 \) is the time at which the last pulse ended, and \( m_0, h_0, \) and \( n_0 \) are the values of \( m, h, \) and \( n \) at that time.

During the pulse, the rate constants are given by:

\[
m(t) = 1 + (m_0 - 1) \exp[-\alpha_M(t-t_0)] \\
h(t) = h_0 \exp[-\beta_H(t-t_0)] \\
n(t) = 1 + (n_0 - 1) \exp[-\alpha_N(t-t_0)].
\]

(2.6)
Hodgkin-Huxley

Pulse-based
Table 1: Conductance-Based Reference Different Methods for Conducting A...
differ from IAF models because they take into account the decay of variables $m$, $h$, and $n$ in between spikes, which has a determinant influence on repetitive firing at high frequencies. In IAF models, the membrane is reset instantaneously after a spike to a hyperpolarized level. In PB models, the reset occurs through the activation of the delayed-rectifier current, which has an effect over a longer period of time due to the relatively slow decay of the variable $n$. This is the main factor that affected the frequency of firing in Figure 2. However, we cannot exclude the possibility that more sophisticated IAF models (with relative refractory period or variable threshold) would reproduce these properties, although it still remains to be demonstrated.

Are PB models different from precalculated conductance waveforms? The model presented here is more sophisticated than simply pasting a conductance waveform, as the changes in variables $m$, $h$, and $n$ depend on their initial values at the time of the spike. Conductance changes are different during low- and high-frequency firing, and PB models propose simplified equations to capture these changes. This is similar to comparing alpha functions with PB models of synaptic currents: Alpha functions do not provide correct postsynaptic summation whereas it is well captured by PB models.
formance of IAF models (see Table 1). However, this increase of efficiency could be minimal in the case of network simulations where action potentials represent only a small fraction of the computation time. On the other hand, if PB models could be used in conjunction with pulse-based mechanisms...