Comment on "Criticality Between Cortical States"

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Fontenele and collaborators [1] reported extensive analyses of power-law scalings of neuronal avalanches in experimental recordings and in a critical theoretical model. One of their main conclusions is that the brain operates at criticality in specific regimes. To prove this point, they developed a test (hereafter, *Fontenele test*) for criticality, which they indicate to rely on our own theoretical result in [2]. However, the results we established in [2] do not provide necessary and sufficient conditions for criticality, and cannot be extrapolated to derive a test for criticality. Moreover, the authors in [1] did not include any control applying the Fontenele test to non-critical systems to assess possible differences with the experimental data. We propose here two such controls, and show that the Fontenele test can misclassify as critical these non-critical systems. This implies that the analyses in [1] do not support their conclusions, and the question of whether the brain operates at criticality remains open.

In detail, the authors conclude that the neuronal systems they study operate at criticality as soon as the data is consistent with *Sethna's crackling relationship* [3]

$$\frac{\tau_t - 1}{\tau - 1} = a$$

where τ_t (resp. τ) are power-law exponents of the *tail* of avalanches duration (resp., size) and a the scaling of the average avalanche size as a function of duration. While this result was shown universal for crackling noise systems, we are not aware of results showing that this relationship implies criticality for neuronal systems, or that neuronal networks belong to the universal class of crackling systems [3]. We have however shown in [2] that two particular non-critical systems do not satisfy this relationship in the thermodynamic limit. But this does not imply that a system satisfying Sethna's relationship is critical. Actually, replicating the analysis in [1], we show that the Fontenele test classifies two non-critical systems as critical in certain situations.

We avalanches in the Brunel model [4] and a Poisson surrogate [2], two non-critical models, using the methodology proposed in [1]. In detail, we (i) considered the bulk of the avalanche distribution by fitting power laws from the smallest observed avalanche up to a cutoff that we varied [10], (ii) tested that the obtained dataset is better fitted by a power-law than by log-normal distribution (AIC difference as in [1]), and (iii) fitted power-law distributions and checked whether Sethna's relationship was satisfied. We repeated these steps for many spike trains and tested the significance of Sethna's relationship.

We found that all samples considered were always better represented by power-laws than log-normal distributions (step (ii)), and moreover that many counter-examples existed where the truncated dataset does pass the Akaike test and is consistent with Sethna's relationship (Figure 1). A two-sample t-test validated with high statistical significance Sethna's relationship for many combinations of thresholds, and therefore the Fontenele test would classify these non-critical systems as critical. These counter-examples therefore show that the Fontenele test is inefficient to distinguish critical from non-critical systems, and therefore the results reported in [1] do not demonstrate that the brain operates at criticality.
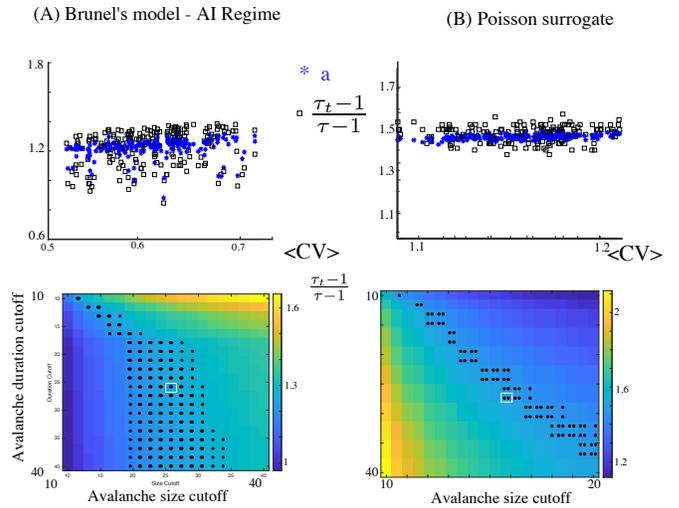

FIG. 1: 200 simulations of Brunel's network [4] (A, parameters as in [2, Fig 7]) and Poisson surrogate (B, Ornstein-Uhlenbeck rate with randomly chosen coefficients), with estimates replicating the analysis in [1]. Bottom: as cutoffs are varied, we observe many combinations where exponents are consistant with Sethna's relationship (2-sample t-test comparing the distribution of ratios and a, *: $p < 0.01$, **: $p < 0.05$, for $n = 14$ instances as in [1]). Top: two examples of cutoffs associated with the white squares in the bottom map.

The systems we presented here serve as counter-examples for the Fontenele test and controls for the study [1]. They are models of the data presented in that paper. More broadly, the literature abounds of counter-examples displaying power-laws in the absence of criticality [6–9]. This underlines the importance for future studies on brain criticality to systematically include control models. Beyond this point, [1] highlights a fascinating similarity between scalings of neuronal behaviors

in vastly distinct species and 'cognitive' states (ex-vivo, slices, anesthetized and behaving). These observations opens enthralling perspectives for investigation on its origin and possible relevance. Theoretical models, both critical *and* non-critical, shall be instrumental to progress in this endeavor. However, to date, the present evidence highlights the fact that the question whether the brain operates at a critical point between cortical states still remains open.

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