The Creutzfeldt-Jakob disease in the Hierarchy of Chaotic Attractors

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1. Introduction

Until recently model co-
2. Phase Portraits of the Creutzfeldt-Jakob Disease

In this section, we shall illustrate the various techniques for the construction of the phase space which will be illustrated with the help of the EEG recorded during the terminal state of the Creutzfeldt-Jakob disease in which it is believed that a virus attacks and destroys gradually the nerve cells. After a first stage of demence, the patient enters in a terminal coma with myoclonus. The EEG is then very coherent (like the "Burst Suppression Pattern" seen after administration of barbiturates) and regular patterns of stable slow waves reminiscent of the "petit mal" type of epileptic seizures appear. However the phenomenon is of much longer duration and shows a remarkable stationarity. Twenty minutes of EEG are shown in Fig. taken from a single lead and twelve of such leads were recorded simultaneously.

The most popular phase space construction results from a theorem shown by Takens. This theorem states that for a dynamical system of \( n \) variables \( X_1 \ldots X_n \), the space spanned by the variables

\[
\{ X_j(t), X_j(t+\tau), X_j(t+2\tau), \ldots, X_j(t+(m-1)\tau) \}
\]
is at least topologically equivalent to the original phase space and therefore represents many of its dynamic properties. Moreover m is restricted by the relation \( m \geq 2d+1 \) where \( d \) is the lowest integer greater than the dimension of the attractor. In this section we refer to such procedures as Takens' construction. Recently it was conjectured that the space spanned by several simultaneous measurements of an observable at various sites of the system is at least topologically equivalent to the original phase space and therefore represents many of its dynamical properties. Moreover m is restricted by the relation \( m \geq 2d+1 \) where \( d \) is the lowest integer greater than the dimension of the attractor. In this section we refer to such procedures as Takens' construction. Recently it was conjectured that the space spanned by several simultaneous measurements of an observable at various sites of the system

\[
\{ X_1(r_1,t), X_1(r_2,t), X_1(r_3,t), \ldots, X_1(r_m,t) \}
\]

may also yield a phase portrait.
3. Dimensional Analysis

Once the variables spanning the phase space become available, the Grassberger & Procaccia algorithm could be used for the evaluation of the correlation dimension $D_2$. We have computed $D_2$ for all four phase portraits of Fig.2. Such a comparative study is in the process of completion for beta waves, alpha wave sleep stages two, four, REM sleep, and "petit mal" epilepsy (Destexhe, Sepulchre & Babloyantz, to be published).

In the case of the Creutzfeldt-Jakob attractor, all four approaches give remarkably comparable results. We report here only the dimensions computed from the usual Takens' construction such as to compare them with the $D_2$ values already reported for other stages of brain activity. If the correlation dimension computed from a single lead (Takens’ construction), we find $D_2 = 3.8 \pm 0.1$. In the case of the Eckman Ruelle construction, the phase space was obtained from 12 EEG leads covering the entire scalp and a value of $D_2 = 3.8 \pm 0.2$ was seen. The concordance between these two values is remarkable.

Although at first sight, the signal appears as extremely coherent with obvious periodicities, we find surprisingly high value for the correlation dimension as compared with the low value of $D_2 = 2.05 \pm 0.1$ observed for the "petit mal" type of epileptic seizure. This finding may stem from the fact that the total phenomenon of the epileptic seizure was of 5 sec. duration whereas the Creutzfeldt-Jakob disease was analyzed over 8 minutes (from a total phenomenon lasting several hours). The analysis of 1 minute recording gave the same result as 8 minutes whereas a 8 sec. time series gave an underestimated dimension of $3.1 \pm 0.2$.

![Graph showing EEG activity](image)

**Figure 3.** Short stretches of some of the most typical episodes of human EEG activity (identically scaled, 12 bit sampling).

In this respect, the Creutzfeldt-Jakob disease may be compared with the normal cardiac activity. At first sight, the electrocardiographic signal appears as periodic however in a recent paper Babloyantz Destexhe have shown that the normal cardiac activity is governed by deterministic chaos characterized by $D_2 = 4.4 \pm 0.4$.

The values of the correlation dimension of Creutzfeldt-Jakob Disease taken together with the $D_2$ computed previously for other stages of brain activity give a coherent image of brain dynamics. The values can be represented as a function of the width of the power spectrum (see Fig.4 and Fig.5). The width is estimated by taking the two extreme frequencies of 75%...
Figure 4. Power spectra of the typical EEG episodes seen in fig.3. The main peaks forming 75% of the spectral energy are underlined with black solid vertical lines. Only for low dimensional systems as (F), the 75% of spectral array is contained relatively few peaks.

Figure 4 shows that the broad spectra correspond to high dimensional attractors such as eyes open and REM sleep. For the brain waves, we found, using the Grassberger & Procaccia algorithm, extreme high values of $D_2$. These values were $D_2 = 9.7 \pm 0.7$ for beta rhythm (Eyes open in fig.3a), $D_2 = 8.2 \pm 0.4$ for REM sleep (Fig.3e). A word of caution is needed here as we are not sure that the algorithm gives reasonable results for such high dimensional systems. In any case, we can say that beta waves at REM sleep behave like colored noise.

In an awake subject with eyes closed, alpha waves set in (Fig.3b). The dynamics becomes more coherent and switches into a deterministic chaotic activity characterized by $D_2 = 6.1 \pm 0.5$ confirmed in other laboratories. As expected, the spectral width also decreases. As the sleep cycle sets in, the brain activity enters a chaotic state of $D_2 = 5.01 \pm 0.03$ for the stage two (Fig.3c) and $D_2 = 4.4 \pm 0.4$ for the stage four or deep sleep (Fig.3d). Similar values were obtained from intra-cranial EEG recordings.
from cats and rabbits. As correlation dimension decreases, so does the width of the spectral band. The sleep stage four is the most coherent stage of the normal brain activity.

In various pathologies, the coherence of the brain dynamics increases further as the correlation dimension decreases. Finally in the "petit mal" type of epileptic seizure (Fig. 3f), the most coherent state reached. Here a near unison is seen in the neuronal activity and the correlation dimension drops to a value of $D_2 = 2.05 \pm 0.09$. Such a low dimensional chaos is seen in three variable differential equations such as the Rössler attractor. As expected, the width of the power spectrum drops dramatically in this case.

![Figure 5](image1.png)

**Figure 5.** Dimension - Power spectrum plot. The extreme frequencies from 75% of the spectral energy are drawn for each EEG episode. Active states are broad banded and high dimensional, while pathologies are of low dimension and their spectrum is restricted to a thin band of frequencies.

![Figure 6](image2.png)

**Figure 6.** Dimension - Amplitude plot. The variations of EEG voltage observed in successive stretches of one second are represented for each type of EEG behavior. The synchronization between neurons which occur in pathologies is reflected in high amplitudes and low dimensions.

Therefore we see from fig. 5 a consistent relationship between the correlation dimension and the maximum energy of the power spectrum. The correlation dimension appears as an increasing function of the spectral width.

The EEG is the sum over a very large number of neuronal membrane potentials and its magnitude therefore a good measure of the synchrony between neurons. Low amplitude EEG reflects relative
desynchronized states whereas high amplitude waves are an indication of synchrony between neural masses. In Fig.6 the correlation dimension of various stages of the brain activity is plotted against measure of the amplitude of the EEG. To obtain this measure, the signal is computed at one second intervals. The average value of these local amplitudes is represented by points in Fig.6. The highest amplitudes are included in the HMM states.
Therefore they are endowed with great information processing capability \(^{19}\), a most conspicuous fact of the cerebral activity.

We believe that with all the problems raised regarding the application of the algorithms, the dimension analysis is a powerful tool for the study of complex biological phenomenon which cannot be handled otherwise. It is especially valuable when it is used as a comparative study as in the case in this paper which follows the evolution of the coherence of the brain waves in various stages of the cerebral activity. The various attractors appear to follow a well defined hierarchy where some key properties, such as the EEG amplitude, the spectral width and the correlation dimension, seem to be intimately correlated. It is expected that this hierarchy could be reproduced by models describing neural networks (work in progress).

We thank Dr. P. Tugendhaft for data acquisition and J.A. Sepulcre for his computer help and for interesting discussions.

A.D. is a fellow from the Institut pour l'Encouragement de la Recherche Scientifique dans l'Industrie et l'Agriculture.

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